Linear codes and Steiner triple systems

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Abstract

Doyen, Hubaut, and Vandensavel [2] proved that the p-rank of the incidence matrix of a Steiner triple system $STS(v)$ on v points can be smaller than v only if $p = 2$ or $p = 3$. Assmus [1] proved that the block by point incidence matrices of all Steiner triple systems on v points which have the same 2-rank generate equivalent binary linear codes, and gave an explicit description of a generator matrix for such a code. The results from [1] were used in [5] to derive an exact formula for the number of distinct $STS(2^n - 1)$ having 2-rank $2^n - n$. The subject of this talk is an alternative, considerably simpler proof of the theorem of Assmus that employs parity check matrices of the relevant codes and allows to prove an analogous result for the ternary linear codes spanned by the block by point incidence matrices of Steiner triple systems [3]. The results from [3] were employed in [4] to derive formulas for the number of distinct $STS(2^n - 1)$ with 2-rank smaller than v, as well as formulas for the number of distinct $STS(3^n)$ having 3-rank smaller than $v - 1$.

References

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