Workshop
Linear programming for energy and designs
Bilateral project KP-06-Austria/8-2025

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ABSTRACTS

Christoph Aistleitner

Equidistribution and local statistics: on the interface of analysis, number theory, and probability

I will introduce some of my research topics, covering in particular equidistribution (uniform distribution modulo one, discrepancy) and local statistics (such as pair correlation and number variance) for sequences of arithmetic origin. A common thread here is the desire to distinguish between "random" (or more precisely, pseudo-random) behavior of deterministic sequences, in contrast to behavior which deviates significantly from the typical behavior of the suitable random model. On a technical level, recurring themes are continued fraction expansions, moment estimates, the combination of analytic and probabilistic methods, and metric entropy.

Peter Boyvalenkov

On optimal T-avoiding spherical codes and designs in $\mathbb{R}^{32}$

we show that the minimal vectors of the extremal even unimodular lattices in $\mathbb{R}^{32}$ are T-avoiding maximal spherical codes for appropriately chosen open set T. Moreover, they are minimal T-avoiding spherical 7-designs, again for suitable T. Five codes are special as they come from lattices generated by the five extremal self-dual codes of length 32 (classified by Conway and Pless in 1980).

Johann Brauchart

Discrepancy of constructed point sets and Energy bounds improvements

The spherical cap L_2-discrepancy of an N-point set measures the deviation from uniform distribution on the unit sphere and can be directly computed using the sum of all mutual distances of the points by means of Stolarsky's invariance principle. Thus, maximal sum of distance points have smallest spherical cap L_2-discrepancy. For the 2-sphere in $\mathbb{R}^3$ the optimal rate is $N^{-3/4}$. The best bound for non-optimal point sets is still the $N^{-1/2}$ rate from a 2012 paper (jointly with Christoph Aistleitner and Josef Dick). This rate is the same as for i.i.d. uniformly distributed points on the sphere. We are interested in the spherical cap L_2-discrepancy of constructed point sets, in particular spherical Fibonacci points obtained by mapping the Fibonacci lattice in the unit square to the sphere using the Lambert transformation.

This is joint work with Josef Dick (UNSW) and Yuan Xu (University of Oregon).

Some of the ideas above provide a starting point to reprove and improve the lower bound for the spherical cap L_2-discrepancy in a joint paper with Dmitriy Bilyk (University of Minnesota), arXiv.2502.15984.

Inspired by this work we could also improve logarithmic energy bounds and Riesz s-energy bounds in the continuous and singular case.

Danila Cherkashin

Linear programming inside Johnson scheme

Let us define *Johnson graph* $J(n, k, t)$, whose vertices are k-element subsets of an n-element set and edges connect pairs of vertices with intersection t. An *independent set* is a vertex subset of a graph such that it does not contain an edge. Let $\alpha(G)$ be the size of a maximal independent set in a graph G. In this language the statement of the Erdős–Ko—Rado theorem is

$$\alpha(J[n,k,0]) = \binom{n-1}{k-1}$$

for $n \geq 2k$. The problem of finding $\alpha(J[n,k,t])$ is known as Erdős–Sós forbidden intersection problem.

We prove that for every $k > 1$ one has

$$\alpha(J[k^2-k+1,k,1]) = \binom{k^2-k-1}{k-2}.$$

The proof uses Linear Programming (Hoffman bound) applied to Johnson association scheme. Seems that it is the only possible tight result that can be obtained in this manner without additional ideas.

Peter Grabner

Hyperuniformity and Energy on Projective Spaces

We study Riesz, Green and logarithmic energy on two-point homogeneous spaces. More precisely we consider the real, the complex, the quaternionic and the Cayley projective spaces. For each of these spaces we provide upper estimates for the mentioned energies using determinantal point processes. Moreover, we determine lower bounds for these energies. Furthermore, we extend the notion of hyperuniformity to the projective spaces and study the connection between energy and the Wasserstein distance.

Maryna Manskova

Hadamard gaps, prime gaps and correlation structures

In this talk I will give an overview of some of my research interests. I will begin with the correlation structures of determinantal point processes, highlighting their characteristic repulsion. Next I will discuss probabilistic aspects of lacunary sums satisfying Hadamard gap condition. I will also mention analytic number-theoretic questions concerning gaps between primes in certain sets.

Maya Stoyanova

Universal Bounds on Energy and Polarization of Weighted Spherical Codes and Designs

For a general absolutely monotone function $h : [-1,1) \to \mathbb{R}$ we obtain universal bounds on the h-energy of weighted spherical codes via linear programming. The universality is in the sense of Cohn–Kumar — every attaining code is optimal with respect to a large class of potential functions (absolutely monotone), in the sense of Levenshtein — there is a bound for every weighted code, and in the sense of parameters (nodes and weights) — they are independent of the potential function. We also derive a universal bound on the minimum of the discrete potential of weighted spherical designs.

Peter Dragnev

Universal polar dual pairs in E_8 and Λ_{24}

We identify universal polar dual pairs of spherical codes C and D such that for a large class of potential functions h the minima of the discrete h-potential of C on the sphere occur at the points of D and vice versa. Moreover, the minimal values of their normalized potentials are equal. These codes arise from the known sharp codes embedded in the even unimodular extremal lattices E_8 and Λ_{24} (Leech lattice). This embedding allows us to use the lattices' properties to find new universal polar dual pairs. In the process we extensively utilize the interplay between the binary Golay codes and the Leech lattice.

As a byproduct of our analysis, we identify a new universally optimal (in the sense of energy) code in the projective space $\mathbb{RP}^{21}$ with 1408 points (lines). Furthermore, we extend the Delsarte-Goethals-Seidel definition of derived codes from their seminal 1977 paper and generalize their Theorem 8.2 to show that if a τ-design is enclosed in $k \leq \tau$ parallel hyperplanes, then each of the hyperplane's sub-code is a $(\tau+1-k)$-design in the ambient subspace.