DECOMPOSITION OF TSCHIRNHAUSEN MODULES FOR COVERINGS ON DECOMPOSABLE \mathbb{P}^1 -BUNDLES

ABSTRACT

Let $\varphi: X \to Y$ be a finite morphism of degree $m \ge 2$, where X and Y are smooth projective algebraic varieties. Such a covering gives rise to the short exact sequence of vector bundles on Y:

$$0 \to \mathcal{O}_Y \xrightarrow{\varphi^{\sharp}} \varphi_* \mathcal{O}_X \to \mathcal{E}^{\vee} \to 0$$
,

where \mathcal{E}^{\vee} is known as the *Tschirnhausen module* the covering φ .

In the talk, we focus on the case where Y is smooth and X is a smooth m-multisection of the \mathbb{P}^1 -bundle

$$f: \mathbb{P}(\mathcal{O}_Y \oplus \mathcal{O}_Y(E)) \longrightarrow Y,$$

with E an effective divisor on Y such that $H^1(Y, \mathcal{O}_Y(kE)) = 0$ for all $k = 1, \dots, m-1$. We show that the Tschirnhausen module of the induced covering $f|_X : X \longrightarrow Y$ is completely decomposable, after which we discuss applications of this result.

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