Hadamard matrices with automorphisms of prime order and related codes

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Abstract

It is known that for every odd prime p there exists a Hadamard matrix of order $n = 2p + 2$ with an automorphism of order p, found by Paley [5], and known in the combinatorial literature as the Paley-Hadamard matrix of type II. If p is an odd prime such that $p \equiv -1 \pmod 3$ then the Paley-Hadamard matrix of type II of order $2p + 2$ is a generator matrix of a Pless symmetry code [6], being a ternary self-dual code of length $2p + 2$. The Pless symmetry codes of length $2p+2$ for $p = 5, 11, 17, 23$ and 29 are ternary extremal self-dual codes that support combinatorial 5-designs. The Paley-Hadamard matrices of type II for $p = 3$ and $p = 5$ are the unique, up to equivalence, Hadamard matrices of orders 8 and 12, respectively. The Hadamard matrices of order $2p + 2$ with an automorphism of order $p = 7, 11, 13, 17, 19$ and 23 have been previously classified up to equivalence [2, 3, 4, 7, 8]. The topic of this talk is the recent classification of Hadamard matrices of order $n = 2p + 2$ with an automorphism of order p for the next two primes, $p = 29$ and $p = 31$ [1]. Up to equivalence, there are 266 Hadamard matrices of order 60 with an automorphism of order 29, and 414 Hadamard matrices of order 64 with an automorphism of order 31. The ternary self-dual codes spanned by the newly found Hadamard matrices of order 60 with an automorphism of order 29 are computed, as well as the binary doubly even self-dual codes of length 120 with generator matrices defined by related Hadamard designs. Several new ternary near-extremal self-dual codes, as well as binary near-extremal doubly even self-dual codes with previously unknown weight distributions are found.

References

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