Anti-concentration of Rademacher sums and Tomaszewski's counterpart problem

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Abstract

A Rademacher sum is a finite weighted sum of independent Rademacher random variables, that is, $R = a_1\epsilon_1 + \cdots + a_n\epsilon_n$ where the a_i are real constants and ϵ_i are picked independently and uniformly at random from $\{-1, +1\}$. Tomaszewski's conjecture from 1986 states that every Rademacher sum R of variance 1 satisfies $\mathbb{P}(|R| \leq 1) \geq 1/2$. This problem has attracted significant attention over the years before it was finally settled by Keller and Klein in 2020. Tomaszewski's counterpart problem is concerned with determining $\inf \mathbb{P}(|R| \geq 1)$, where the infimum is taken over the class of Rademacher sums R of variance 1. This problem has also received much attention over the years, with Hitczenko and Kwapień conjecturing in 1994 that the infimum is 7/32. We confirm Hitczenko and Kwapień's conjecture. Our methods enable us to fully determine $f(y) = \inf \mathbb{P}(|R| \geq y\sqrt{\mathrm{Var}(R)})$ where the infimum is taken over all Rademacher sums R, confirming a conjecture by Lowther and giving a partial answer to a question by Keller and Klein. This is joint work with Lawrence Hollom.