Introduction to tensor triangular geometry

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Tensor triangular geometry is the study of tensor triangulated categories by algebro-geometric methods. It is a geometric approach to certain classification problems in a variety of contexts such as algebraic geometry, stable homotopy theory, modular representation theory and more. In each of these contexts one is presented with a familiar triangulated category that comes with extra structure in the form of a symmetric monoidal product. Using this extra structure, Balmer has introduced the notion of a spectrum of a tensor triangulated category, a construction analogous to the construction of the spectrum of an ordinary commutative ring. It turns out that determining this spectrum is equivalent to the classification of thick triangulated tensor ideals of the category in question. This paves the way for the concept of *tt-classification* or *classification up to the tensor-triangular structure*. The theory allows one to decide whether or not two objects X and Y of a tensor triangulated category $\mathcal{K}$ are the same modulo the basic operations available in $\mathcal{K}$: cones, direct summands, tensor products. One of the surprising results of this machinery, that may appeal to the algebro-geometric audience, is that one can completely reconstruct a quasi-compact quasi-separated scheme as the Balmer spectrum of its bounded derived category of perfect complexes.

In a series of four talks, I will try to give a brief introduction to the subject and look at examples and applications following the exposition of [3] and [1]. The main references on the subject are [2] and [4].

A preliminary list of the topics I will aim to cover is as follows:

1. Triangulated categories and localization
2. Tensor triangulated categories and the Balmer spectrum
3. Universality, functoriality and localization
4. Examples and applications

References

[1] Paul Balmer. "A guide to tensor-triangular classification". In: *Handbook of Homotopy Theory*. Ed. by Haynes Miller. 2019.

[2] Paul Balmer. "Presheaves of triangulated categories and reconstruction of schemes". In: *Mathematische Annalen* 324 (Nov. 2002), pp. 557–580.

[3] Paul Balmer. "Tensor Triangular Geometry". In: *Proceedings of the International Congress of Mathematicians 2010 (ICM 2010)*, pp. 85–112.

[4] Paul Balmer. "The spectrum of prime ideals in tensor triangulated categories". In: *Journal fur die Reine und Angewandte Mathematik* 2005 (Oct. 2004).