СЕКЦИЯ

„АЛГЕБРА И ЛОГИКА”

Драги колеги,
На 23 юни 2023 г. (петък) от 13:00 часа в зала 578 на ИМИ-БАН ще се проведе присъствено заседание на семинара по „Алгебра и логика”.
Доклад на тема

Distinctness of the “Lifted” Kloosterman Sums over the Prime Field $\mathbb{F}_p$

ще изнесе Любомир Борисов.

Поканват се всички желаещи да присъстват.

От секция „Алгебра и логика” на ИМИ – БАН

http://www.math.bas.bg/algebra/seminarAiL/

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DISTINCTNESS OF THE "LIFTED" KLOOSTERMAN SUMS OVER THE PRIME FIELD $\mathbb{F}_p$

LYUBOMIR BORISSOV

Abstract. In this talk I consider the Kloosterman sums over the finite field $\mathbb{F}_q$ of characteristic p, defined by

$$\mathcal{K}_q(u) = \sum_{x \in \mathbb{F}_q^*} \omega^{\,Tr(x+ux^{-1})},$$

where $\omega = e^{\frac{2\pi i}{p}}$ is a primitive $p-$th root of unity, and $Tr(a)$ is the absolute trace of $a \in \mathbb{F}_q$ over $\mathbb{F}_p$.

The focus of special attention are the so-called "lifted" Kloosterman sums over $\mathbb{F}_q$ (see, [2]), i.e., $\mathcal{K}_{q^n}(u), u \in \mathbb{F}_q$, where $\mathbb{F}_{q^n}$ is the finite field of order $q^n, n > 1$.

It is well-known that the Kloosterman sums play an important role in algebraic coding theory and cryptography (see, e.g., the surveys [3]-[4]).

Firstly I clashed with them in the problem of enumerating the elements of a finite field having prescribed trace and co-trace:

https://arxiv.org/pdf/1711.08306.pdf

The issue of their distinctness is considered and partly solved for the first time by Benjamin Fisher in 1992 [5]. In particular, this author has proved that fact for the simplest sums, i.e., over the prime fields.

Recently, in a personal communication with us, Daqing Wan has announced that as a co-product of his research [6] (based on deep algebraic number theory such as Stickelberger's theorem) it follows the distinctness of "lifted" Kloosterman sums over any prime field $\mathbb{F}_p$ whenever the extension degree is not a multiple of p. This statement generalizes our result for the fields whose extension degree is a power of 2:

https://link.springer.com/article/10.1007/s12095-020-00443-1

The case $p = 3$ I considered in [1]. Here I give a complete proof that all "lifted" Kloosterman sums over each prime field of characteristic $p > 3$ and any extension degree, are distinct.

References

[1] ON THE DISTINCTNESS OF SOME TERNARY KLOOSTERMAN SUMS Lyubomir Borissov Proceedings of the Fiftieth Spring Conference of the Union of Bulgarian Mathematicians, (2021)

[2] L. Carlitz, "Kloosterman sums and finite field extensions", Acta Arithmetika vol. XVI.2 (1969), pp. 179-193.

[3] N. E. Hurt, "Exponential sums and coding theory: a review", Acta Appl. Math., vol. 46.1 (1997), pp. 49-91.

[4] V. A. Zinoviev, "On classical Kloosterman sums", Cryptogr. and Commun., 11.3 (2019), pp. 461-496.

[5] B. Fischer, "Distinctness of Kloosterman sums", Contemporary Mathematics, vol. 133 (1992), pp. 81-102.

[6] D. Wan, "Minimal polynomials and distinctness of Kloosterman sums", Finite Fields Appl., 1 (1995), pp. 189-203.

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