Jordan algebra conformal toolbox

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We employ the Jordan algebras for a succinct description of the dynamical conformal symmetries of integrable models. Given an Euclidean Jordan algebra $\mathfrak{J}$ via Tits-Kantor-Köcher construction we obtain a representation of the conformal (Möbius) group $Co(\mathfrak{J})$. Since the seminal work of Gerhard Mack and Ivan Todorov [1] on irreducible minimal conformal group $U(2,2)$ representation we know the orbital wavefunctions of the hydrogen atom live in a minimal $U(2,2)$-representation. Given the Jordan algebra $H_2(2,\mathbb{C})$ of hermitian 2×2 matrices (Pauli matrices)[2] we recover the hydrogen spectrum $U(2,2)$-representation from the TKK construction via $U(2,2)\cong Co(H_2(2,\mathbb{C}))$. A reality condition imposed on the Jordan algebra of Pauli matrices yields $H_2(2,\mathbb{R})$ and reduces the 3D H-atom to a 2D system. The Majorana reduction of the Dirac spinor transforming under $U(2,2)$ yields the dynamical conformal symmetry $Sp(4,\mathbb{R})$ of the quantum motion of an electron in magnetic field (Landau problem). Different Landau levels turn out to be packed into a single conformal spinorial representation of $SO(3,2)$ which is identified with the Dirac's "Remarkable representation of the 3+2 de Sitter group"[3]. We finally speculate on higher Jordan algebras and their relevance to the mass spectrum of elementary particles [4].

References

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