Variations of GIT-quotients of flag varieties

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Abstract: Let G be a semisimple simply connected complex Lie group, $B \subset G$ a Borel subgroup and $X = G/B$ the complete flag variety of G. The Picard group of X is isomorphic to the weight lattice Λ of a maximal torus $T \subset G$. The effective line bundles correspond to the dominant weights Λ^+ and their section spaces are models for the irreducible G-modules. The Cox ring of X contains exactly one copy of each irreducible module.

Now, given a reductive subgroup $L \subset G$, one can consider two natural problems: 1) describe the L-orbits in X; 2) describe the decompositions of irreducible G-modules under L. The Geometric Invariant Theory (GIT) of Hilbert-Mumford provides a relation between these problems and some theoretic methods for investigation.

In this talk, partly based on joint work with H. Seppänen, I will present a description of the GIT-classes of L-ample line bundles on X and some properties of the respective GIT-quotients. Under mild assumptions, we prove the existence of a quotient whose Cox ring is, up to a finite extension, isomorphic to the ring of L-invariants in the Cox ring of X. This is indeed a special property, as such a quotient inherits, a priori, only information about the ample line bundle with respect to which it is defined.