On the minimal sum of edges in a signed edge-dominated graph

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on a joint work with P. Prozorov

Let G be a graph; for a given edge $e = (u, v)$ define its *closed edge-neighborhood* as an edge subset $N[e]$ formed by e and all edges of G adjacent to e. A weight function $f : E \to \{+1; -1\}$ is called a *signed edge domination function* of G if

$$\sum_{e' \in N[e]} f(e') \geqslant 1$$

for every $e \in E$; we say that (G, f) is an *SED-pair* of order $|V|$.

The following problem was posed by Xu in [2]. What is

$$g(n) := \min\{s[(G, f)] \,|\, (G, f) \text{ is an SED-pair of order } n\}$$

for each positive integer n?

Since a blow-up preserves the signed edge domination property, one may deduce that

$$g(n) = (1 + o(1))Hn^2$$

for some negative constant H.

We refine [1] known upper and lower bound on H, id est

$$-\frac{1}{25} \leqslant H \leqslant -\frac{1}{8(1+\sqrt{2})^2}.$$

It turns out that the problem quickly reveals its analytical nature. At the moment we have a deal with a multivariate polynomial optimization problem, which we are unable to solve. On the other hand it definitely refines the current lower bound on H.

References

[1] Danila Cherkashin and Pavel Prozorov. On the minimal sum of edges in a signed edge-dominated graph. *The Electronic Journal of Combinatorics*, 29(3):P3.38, 2022.

[2] Baogen Xu. On signed edge domination numbers of graphs. *Discrete Mathematics*, 239(1-3):179–189, 2001.