

# СЕКЦИЯ

## „АЛГЕБРА И ЛОГИКА”

Драги колеги,

На 11 февруари 2022 г. (петък) от 13:00 часа ще се проведе дистанционно заседание на семинара по „Алгебра и логика”.

Доклад на тема

### Modal definability of some classes of modal products

ще изнесе Яна Руменова (ФМИ – СУ).

Докладът е съвместен с Тинко Тинчев (ФМИ – СУ).

Семинарът ще се проведе посредством платформата **Zoom** и всеки желаещ може да се присъедини като последва линка, зададен на страницата на семинара.

От секция „Алгебра и логика” на ИМИ – БАН

<http://www.math.bas.bg/algebra/seminarAiL/>

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#### Abstract

Let  $\mathcal{K}_{commute}$  be the class of structures for the language  $\mathcal{L}(R_1, R_2, \dot{=})$  in which the two equivalence relations commute and let  $\mathcal{K}_{partition}$  be the class of structures for the language  $\mathcal{L}(R, \dot{=})$  of all partitions.

In this talk we will review the decidability of first-order theories of the following subclasses of  $\mathcal{K}_{commute}$ :

- Let for each  $n \in \omega^+$   $\mathcal{K}_{commute}^{R_1 \leq n}$  be the class of all structures from  $\mathcal{K}_{commute}$  such that for each matrix in the structure the rows have  $\leq n$  number of cells (we shall introduce these notions later on);

- Let for each  $n \in \omega^+$   $\mathcal{K}_{commute}^{R_1 \leq n, R_2 < \omega}$  be the class of all structures from  $\mathcal{K}_{commute}^{R_1 \leq n}$  such that for each matrix in the structure the columns have a finite number of cells;
- Let for each  $n, m \in \omega^+$  be the tighter subclass of  $\mathcal{K}_{commute}^{R_1 \leq n, R_2 < \omega}$ :  $\mathcal{K}_{commute}^{R_1 \leq n, R_2 \leq m}$ ;
- Let  $\mathcal{K}_{rectangle}$  be a subclass of  $\mathcal{K}_{commute}$  such that the structures are modal products of structures from  $\mathcal{K}_{partition}$ ;
- Let  $\mathcal{K}_{square}$  be a subclass of  $\mathcal{K}_{rectangle}$  such that the structures are modal products of a structure from  $\mathcal{K}_{partition}$ .

Because  $\mathcal{K}_{commute}^{R_2 \leq n}$  is similar to  $\mathcal{K}_{commute}^{R_1 \leq n}$  and  $\mathcal{K}_{commute}^{R_2 \leq n, R_1 < \omega}$  is similar to  $\mathcal{K}_{commute}^{R_1 \leq n, R_2 < \omega}$ , we will only discuss FMP and the decidability problem of  $\mathcal{K}_{commute}^{R_1 \leq n}$  and  $\mathcal{K}_{commute}^{R_1 \leq n, R_2 < \omega}$ . The same reasoning can be applied for obtaining the results for the other two classes.

We use methods from general/finite model theory like Ehrenfeucht-Fraïssé games and results on generalized products started by Mostowski and continued by Feferman and Vaught to demonstrate decidabilities of the first-order validity problems and the possession of the finite model property.

These classes almost fit the criteria of the definition of a stable class conjured by Balbiani and Tinchev, so we will call them pre-stable.

We will also show that the problem of deciding the validity of sentences in each of these subclasses of  $\mathcal{K}_{commute}$  is reducible to the modal definability problem w.r.t. the subclass in question. Alas, this only gives us a lower bound of the complexity of the modal decidability problems w.r.t. each of these classes.