Extremes of Markov random fields on block graphs

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Abstract

A graphical model is a random vector with a distribution containing a set of conditional independence relations which can be represented by a graph of nodes and edges: on every node there is a variable and an edge represents conditional dependence between two variables. We attempt to study limits of extremes of such models.

In particular we study the joint occurrence of large values of a Markov random field or undirected graphical model associated to a block graph. On such graphs, containing trees as special cases, we aim to generalize recent results for extremes of Markov trees (Segers, 2020). Formally, the limit of

$$(X_v/X_u)_{v=1,\dots,d} \mid X_u > t, \qquad t \to \infty.$$

depends on the unique shortest paths between two variables on the graph. Potential application of the models are extremes on river networks. The latter ressemble more a tree, a special case of a block graph.

When the sub-vectors induced by the blocks follow Hüsler–Reiss extreme value copulas, the global Markov property of the original field induces a particular structure on the parameter matrix of the limiting max-stable Hüsler–Reiss distribution. The multivariate Pareto version of the latter turns out to be an extremal graphical model in the sense of Engelke and Hitz (2020).

Another line of research when considering a particular parametric model is the problem of identifiability of all dependence parameters if some variables are latent. Latent (unobserved) variables occur often in applications related to river networks.

References

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