

Annual Reporting Session of the Department of Algebra and Logic
December 18, 2020
Abstracts of the Talks

Vesselin Drensky

New examples of fundamental algebras

(Joint project with Luisa Carini, University of Messina, Italy)

Fundamental algebras are the building blocks used to generate any variety of finite basic rank of associative algebras over a field of characteristic 0. Our first result describes the fundamental algebras which are tensor products of any number of finite dimensional Grassmann algebras. Then we show that the triangular product of two fundamental algebras is again fundamental. The proofs are based on the recent description of fundamental algebras in the language of cocharacter sequences due to Giambruno, Polcino Milies and Zaicev combined with other techniques from the theory of PI-algebras.

Vassil Kanev

Hurwitz modular varieties parametrizing coverings with fixed monodromy group

Given a projective curve Y , a transitive subgroup G of the symmetric group S_d and a natural number n the talk is devoted to smooth families of coverings of Y of degree d branched in n points whose monodromy group is G . These families form a category whose morphisms correspond to the pullback by morphisms of the bases of the families. Under certain restrictions on the group G we construct a universal family in this category. We discuss how to change the category, so that the universal family exists without any restrictions on G .

Ivan Chipchakov

Fields of dimension one, normal over a global or local field

Let E be a normal extension of a global or local field K . We show that if K is a local field, v is its (natural) discrete valuation, and q is the characteristic of the residue field of (K, v) , then E is a field of dimension $\dim(E) \leq 1$ if and only if the following conditions hold: for each prime number $p \neq q$, E contains as a subfield an unramified \mathbb{Z}_p -extension K_p of K ; the restriction $p \neq q$ is dropped in case the value group $v'(E)$ is q -in divisible, where v' is the unique, up-to equivalence, valuation of E extending v . When K is a global field and E/K is abelian and tamely ramified, nontrivial Krull valuations of E are discrete; also, $\dim(E) \leq 1$ if and only if E is a nonreal field and the residue fields of these valuations are algebraically closed. Under the hypothesis that K is a global field, E/K is abelian and $\dim(E) \leq 1$, this is used for proving the existence, for each $n \in \mathbb{N}$, of n -variate homogeneous polynomials of degree n with coefficients in E , which violate the local-to global principle over E .

Tatiana Gateva-Ivanova

Associative algebras and Lie algebras defined by Lyndon words

Assume that $X = \{x_1, \dots, x_g\}$ is a finite alphabet and \mathbf{k} is a field. We study the class $C(X;W)$ of associative graded \mathbf{k} -algebras A generated by X and with a fixed obstructions set W consisting of Lyndon words in the alphabet X . Important examples are the monomial algebras $A = \mathbf{k}\langle X \rangle / (W)$, where W is an antichain of Lyndon words of arbitrary cardinality and the enveloping algebra $U\mathfrak{g}$ of any X -generated Lie \mathbf{k} -algebra $\mathfrak{g} = \text{Lie}(X) = ([W])$, whenever the set of standard bracketings $[W] = \{[w] \mid w \in W\}$ is a Gröbner-Shirshov Lie basis. We prove that all algebras A in $C(X;W)$ share the same Poincare-Birkhoff-Witt type \mathbf{k} -basis built out of the so called *Lyndon atoms* N (determined uniquely by W) but, in general, N may be infinite. Moreover, A has polynomial growth if and only if the set of Lyndon atoms N is finite. In this case A has a \mathbf{k} -basis $N = \{l_1^{\alpha_1} \dots l_d^{\alpha_d} \mid \alpha_i \geq 0, 1 \leq i \leq d\}$, where $N = \{l_1, \dots, l_d\}$. Surprisingly, in the case when A has polynomial growth its global dimension does not depend on the shape of its defining relations but only on the set of obstructions W : We prove that if A has polynomial growth of degree d then A has global dimension d and is standard finitely presented, with $d-1 \leq |W| \leq d(d-1)/2$. We study when the set of standard $[W] = \{[w] \mid w \in W\}$ is a Gröbner-Shirshov Lie basis. We use our general results to classify the Artin-Schelter regular algebras A generated by two elements, with defining relations $[W]$ and global dimension ≤ 7 .

References

- [1] Tatiana Gateva-Ivanova, *Algebras defined by Lyndon words and Artin-Schelter regularity*, To appear in The Transactions AMS arXiv preprint arXiv:1905.11281 (2019).
- [2] Tatiana Gateva-Ivanova, Gunnar Floystad, *Monomial algebras defined by Lyndon words*, Journal of Algebra **403** (2014), 470{496.
- [3] Tatiana Gateva-Ivanova, *Quadratic algebras, Yang-Baxter equation, and Artin-Schelter regularity*, Advances in Mathematics **230** (2012), 2152{2175.
- [4] Tatiana Gateva-Ivanova, *Global dimension of associative algebras*, Applied Algebra, Algebraic Algorithms and Error-Correcting Codes, Lecture Notes in Computer Science, **357** (1989), 213-229.

Jörg Koppitz

The generators of the semigroup of all transformations preserving a crown

In this presentation, we will give a survey about the status of the study of monoids of transformations preserving a fence and a crown, respectively. In particular, we will consider the monoid of automorphisms preserving fence and crown, respectively. Finally we will give an idea of the current status of study of the rank of the monoid of all partial transformations preserving a finite crown.

Dimitrinka Vladeva

Derivations of skew Ore polynomial semirings

In this project we investigate derivations in the semiring of skew Ore polynomials over an additively idempotent semiring. We show that multiplying each polynomial by x on left is a derivation and construct commutative idempotent semiring consisting of derivations of a skew polynomial semiring. We introduce hereditary derivations and generalized hereditary derivations defined as derivations acting only over the coefficients of the polynomial and also construct an S -derivation in the classical sense of Jacobson. Finally we give a description of the derivations in a skew polynomial semiring $S[x]$, assuming that S is an additively idempotent semiring and show that an arbitrary derivation can be represented by a generalized hereditary derivation and an S -derivation.

Vladimir S. Gerdjikov

Recursion operators and the hierarchies of MKdV equations related to $D_4^{(1)}$, $D_4^{(2)}$ and $D_4^{(3)}$ Kac-Moody algebras

Reference:

V. S. Gerdjikov, A.A. Stefanov, I. D. Iliev, G. P. Boyadjiev et al. Recursion operators and the hierarchies of MKdV equations related to $D_4^{(1)}$, $D_4^{(2)}$ and $D_4^{(3)}$ Kac-Moody algebras. *Theoretical and Mathematical Physics*, **204** (3): 1110–1129 (2020), ArXiv:2006.16323 [nlin.SI]

Elitza Hristova

Regularity of algebras of $O(n)$ -invariants using Hilbert series

Let W be a polynomial representation of the complex general linear group $GL(n)$. In this talk, we discuss the question when the algebra of invariants $\mathbb{C}[W]^{O(n)}$ is regular, i.e. isomorphic to a polynomial algebra. For $n=2$, we give a list of polynomial $GL(2)$ -representations, so that if $\mathbb{C}[W]^{O(2)}$ is regular, then up to an $O(2)$ -isomorphism W is in this list. For general n , we prove regularity in particular cases. The talk is based on a joint work with Vesselin Drensky.

Veselin Filev

Holographic Berkooz-Douglas Matrix Model at Finite Temperature

I will report on ongoing work to construct the holographic dual supergravity background of the Berkooz-Douglas matrix model at finite temperature.

Vilislav Buchackchiev

Forecasting of the Bulgarian House price index using some macroeconomic indicators

With the introduction of IFRS9 accounting standard in 2018 many banks were required to use statistical models for forecasting the liquidation values of houses used as collateral for mortgages. The nature of estimation of the expected credit loss requires the evaluation of levels of House Price Index from available statistical data which is, usually, one year old. Several specifications of the models were studied to confirm that HPI is correlated with various indicators, including RE market demand, construction industry business cycle and general macroeconomic environment. The general conclusion was, however, that the two most prominent drivers of HPI remain the interest rates and the internal inertia of the RE market.

Dimiter Dobrev

Language for Description of Worlds

We will reduce the task of creating AI to the task of finding an appropriate language for description of the world. This will not be a programming language because programming languages describe only computable functions, while our language will describe a somewhat broader class of functions. Another specificity of this language will be that the description will consist of separate modules. This will enable us look for the description of the world automatically such that we discover it module after module. Our approach to the creation of this new language will be to start with a particular world and write the description of that particular world. The point is that the language which can describe this particular world will be appropriate for describing any world.

Valentin Iliev

The Uncertainty Principle for Screening Tests

The aim of this elementary note is to describe the relation between the conditional probabilities of a false positive and a false negative screening test. Non-formally, we can state the main result of the paper as an Uncertainty Principle: In general, if one has better knowledge that the test is really positive (the probability F_{+} of false positive test is small), then for one is hard to know that the test is really negative (the probability F_{-} of false negative test is large). And the better one knows that the test is really negative (F_{-} is small), the harder it is to know that the test is really positive (F_{+} is large).

Roussanka Loukanova

Type-Theory of Parametric Algorithms

I shall present a class of Moschovakis type-theories of recursion. My focus is on an overview from the perspective of existing and potential applications. I shall point to some of my contributions on these topics.

Valentin Goranko

Rational coordination in Pure Win-Lose Coordination Games

The main question I will address is: when and how can rational agents coordinate without any prior communication or conventions? I will consider this question in the abstract framework of multi-player pure coordination games, where each player has a number of possible choices, every choice profile determines a unique outcome, and in every outcome all players have identical payoffs, 'win' or 'lose'.

I will formally introduce pure win/lose coordination games and will present and discuss a hierarchy of 'rationality principles' that can be applied by rational players in such games to determine their choices of action. Then I will compare the strength of some of these principles in terms of the classes of coordination games that can be solved by them by using only pure reasoning, without any preplay communication and conventions. I will argue that the boundaries between pure rationality principles and other rational decision methods used for solving coordination games are quite debatable and there is apparently no clear distinction between these.

Lastly, time permitting I will discuss briefly how pure coordination games can be solved with the use of 'structural' conventions (only based on structural properties of the games), agreed in a preplay communication, and will describe precisely the scope of purely rational coordination.

The talk is based on this recent joint paper with Antti Kuusisto and Raine Rönholm:

<https://academic.oup.com/logcom/article/30/6/1183/5869758?guestAccessKey=374b9c38-2900-4302-91c6-8c8c1eac6ac4>.

Dimitar Guelev

Strategy Profiles and a Vocabulary for Solving Infinite Concurrent Games

with Temporary Coalitions in QCTL*

This is a short version of my seminar talk from October 30. It highlights the key notion and notations proposed in that talk. The key notion is a straightforward extension of the notion of strategy profile for registering varying partitionings of the totality of the players into disjoint coalitions. The notation is a vocabulary for the propositionally quantified branching time temporal logic QCTL* which augments the encoding of strategy profiles into this logic as known from the literature, including my previous work, with symbols for specifying shifting coalition structure. In this short presentation, we focus on the notation and sideline the key technical

results of the work, which show that complete information concurrent multiplayer infinite games with LTL-definable partially ordered objectives are solvable wrt whatever solution concepts happen to be expressible in the proposed vocabulary. That includes temporary coalition generalisations of some established solution concepts. The work is available from the 8th International Workshop on Strategic Reasoning and arXiv.