

Infinite-color, randomly reinforced urn processes with dominant colors

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Abstract

In this work we consider a general class of stochastic processes with random reinforcement that are extensions of the celebrated Pólya sequence by Blackwell and MacQueen [*Ann. Stat.* **1** (1973) 353–355]. The resulting randomly reinforced Pólya sequence (RRPS) can be described as an urn scheme with countable number of colors and a general replacement rule. We study a particular specification of the replacement mechanism of a RRPS, which allows for the discrimination of colors based on their expected reinforcement; thus, colors associated to larger reinforcement values on average are deemed dominant. The implied sampling procedure is best understood in the context of randomized, response-adaptive designs of clinical trials for a single treatment, whereby each draw from the urn indicates the delivery of a specific dose from the treatment to a new patient, and the subsequent reinforcement of the urn contents is a function of the patient's response to it. Our results show, in particular, that the conditional probability of administering a dominant dose and the number of patients given a dominant dose both go to 1. In fact, under further restrictions on the reinforcement, the predictive and empirical distributions of the observation process converge in the sense of almost sure weak convergence to one and the same random probability measure, whose mass is concentrated on the dominant set. We provide further central limit results and fit some existing finite-color urn models to our framework.

Keywords: Reinforced processes. Pólya sequence. Randomly reinforced urn. Dominant colors. Response-adaptive design of clinical trials.