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Title: A topological characterization of dual strict convexity in Asplund spaces.

Abstract:

We say that a topological space Z has $(*)$ if there is a sequence $(\mathcal{U}_j)_{j=0}^{\infty}$ of families of open subsets of Z , with the property that given $x, y \in Z$, there exists $j \in \mathbb{N}$ such that

1. $\{x, y\} \cap \bigcup \mathcal{U}_j$ is non-empty, and
2. $\{x, y\} \cap U$ is at most a singleton for all $U \in \mathcal{U}_j$.

This property was introduced by J. Orihuela, S. Troyanski and the author several years ago, in relation to strictly convex norms on Banach spaces.

We show that if X is an Asplund space, then it admits an equivalent norm having a strictly convex dual norm if and only if the dual unit sphere S_{X^*} (equivalently X^*), endowed with the w^* -topology, possesses $(*)$. It follows that this ostensibly geometric property of the space can in fact be characterized in purely non-linear, topological terms. This improves upon a previous characterization, obtained by the authors above, which required an additional linearity assumption.