TAILS AND PROBABILITIES FOR *P*-OUTLIERS

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Abstract. The task for a general and useful classification of the heaviness of the tails of probability distributions still has no satisfactory solution. Due to lack of information outside the range of the data the tails of the distribution should be described via many characteristics. Index of regular variation is a good characteristic, but it puts too many distributions with very different tail behavior in one and the same class. One can consider for example Pareto(α), Fréchet(α) and Hill-horror(α) with one and the same fixed parameter $\alpha > 0$. The main disadvantage of VaR, expectiles, and hazard functions, when we speak about the tails of the distribution, is that they depend on the center of the distribution and on the scaling factor. Therefore they are very appropriate for predicting "big losses", but after a right characterization of the distributional type of "the payoff". When analyzing the heaviness of the tail of the observed distribution we need some characteristic which does not depend on the moments because in the most important cases of the heavy-tailed distributions theoretical moments do not exist and the corresponding empirical moments fluctuate too much. In this paper, we show that probabilities for different types of outliers can be very appropriate characteristics of the heaviness of the tails of the observed distribution. They do not depend on increasing affine transformations and do not need the existence of the moments. The idea origins from Tukey's box plots, and allows us to obtain one and the same characteristic of the heaviness of the tail of the observed distribution within the whole distributional type with respect to all increasing affine transformations. These characteristics answer the question:

At what extent we can observe "unexpected" values?