Convergence of a fourth-order method for simultaneous finding polynomial zeros

Slav I. Cholakov, Milena D. Petkova

Faculty of Mathematics and Informatics, University of Plovdiv
cholakovs@uni-plovdiv.bg, milenapetkova@uni-plovdiv.bg


Let \((\mathbb{K}, |·|)\) denote an arbitrary normed field, \(\mathbb{K}[z]\) denote the ring of polynomials over \(\mathbb{K}\), and let \(f \in \mathbb{K}[z]\) be a polynomial of degree \(n \geq 2\). Petković-Ranšić-Milošević’s method is defined by the following fixed-point iteration in \(\mathbb{K}^n\):

\[
x_{k+1} = T(x_k), \quad k = 0, 1, 2, \ldots,
\]

where the operator \(T: \mathbb{K}^n \to \mathbb{K}^n\) is defined by \(T(x) = (T_1(x), \ldots, T_n(x))\) with

\[
T_i(x) = x_i - \frac{f(x_i)}{f'(x_i)} - \frac{f''(x_i)}{f'(x_i)} \left( \sum_{j \neq i} \frac{1}{x_i - x_j} \right)^2
\]

for \(i = 1, \ldots, n\).

In this talk, we establish a new local convergence theorem with error estimates for this method. In particular, an estimate of the radius of the convergence ball of the method is obtained. Our result is obtained by using some results of [2] and [3].

Keywords: Simultaneous methods, Polynomial zeros, Local convergence, Error estimate

Acknowledgment. The talk is supported by Project NI15-FMI-004 of Plovdiv University.

References

